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On the use of artificial neural networks when solving conservation laws

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The complexity of numerically solving nonlinear hyperbolic conservation laws has lead to the development of modern methods with a variety of nonlinear elements, e.g., slope limiters, nonlinear viscosity, stencil selection, and also Riemann solvers can be characterized as such. While many years of progress has resulted in effective techniques to address these problems, such solutions often remain bottlenecks for either efficiency or overall performance when solving complex problems. Furthermore, many existing techniques eventually rely on the specification of one or several problem-specific parameters, making it difficult and time-consuming to apply such methods in a broader context.

In this talk we discuss how ideas from modern machine learning can be used to address some of these challenges. Focusing primarily on the use of small artificial neural networks, we discuss how to implicitly learn many of the rules that past work attempts to make explicit, e.g., to decide where a local numerical solution is of poor quality and should be marked for limiting or adaptivity. As we shall see, a priori learning these rules in a purely data driven fashion offers a number of advantages and suggests that such an approach could be an path toward improved methods.

After a very brief introduction to artificial neural networks, we discuss examples of how such networks can be trained and used in a collaborative mode within existing modern computation methods for solving conservation laws.

As an example of a classification problem, we discuss the development of troubled cell indicators in high-order methods [1, 2] and demonstrate that this can be achieved with high accuracy, is parameter free, and accomplished with a network that is problem independent.

By discussing a problem which can either be seen as a multi-class classification problem, by estimating local solution regularity, or as a regression problem, by directly estimating a nonlinear viscosity, we shall illustrate the specification of local non-linear viscosity to control numerical oscillations in high-order methods used to solve problems with discontinous solutions [4, 5, 6].

Time permitting, we discuss a final example in which the expensive Riemann solvers is replaced with a suitably trained network, giving the Riemann solution as well as wave-speeds of specific waves [3]. This is an example of a more complex problem as it requires the construction of a network which satisfies certain physical constraints, e.g., mass conservation.

All the techniques that we discuss are general and the trained networks can be used for a variety of problems without having to be retrained or altered.

The different developments will be illustrated by computational results in one- and two-dimensions to demonstrate the efficiency of these techniques which, as we shall likewise discuss, are often both more accurate and faster than more traditional techniques.

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