

Low rank structure in forward and inverse kinetic theory

Qin Li *

Multi-scale kinetic equations can be compressed: In certain regimes, the Boltzmann equation is asymptotically equivalent to the Euler equations, and the radiative transfer equation is asymptotically equivalent to the diffusion equation. Detailed information is lost when a system passes to the fluid limit. In linear algebra, it is equivalent to a system being of low rank. I will discuss such transition and how it affects the computation. Mainly, in the forward regime, inserting low-rank structure could greatly speed up the computation [1, 2], while in the inverse regime, the system being of low rank typically makes the problems significantly more ill-posed [3, 4]. This is a review of a collection work with Ke Chen (UT-Austin), Kathrin Hellmuth (Würzburg), Christian Klingenberg (Würzburg), Ru-Yu Lai (UMN), Jian-feng Lu (Duke), Gunther Uhlmann (UW-Seattle), and Stephen J. Wright (UW-Madison).

Acknowledgements

This research has been partially supported by NSF, ONR, Vilas Early Career, and WARF at UW-Madison.

References

- [1] K. CHEN, Q. LI, J. LU, AND S. J. WRIGHT, *Random sampling and efficient algorithms for multiscale pdes*, SIAM Journal on Scientific Computing, 42 (2020), pp. A2974–A3005.
- [2] ———, *A low-rank schwarz method for radiative transfer equation with heterogeneous scattering coefficient*, Multiscale Modeling & Simulation, 19 (2021), pp. 775–801.
- [3] K. HELLMUTH, C. KLINGENBERG, Q. LI, AND M. TANG, *Multiscale convergence of the inverse problem for chemotaxis in the bayesian setting*, Computation, 9 (2021).
- [4] R.-Y. LAI, Q. LI, AND G. UHLMANN, *Inverse problems for the stationary transport equation in the diffusion scaling*, SIAM Journal on Applied Mathematics, 79 (2019), pp. 2340–2358.

*UW-Madison. Email: qinli@math.wisc.edu